Supplement to the Curriculum Frameworks

Table of Contents

GRADES K – 2	Page
ONADEO N - Z	
NUMBER SENSE	3
GEOMETRIC AND SPATIAL SENSE	25
DATA ANALYSIS, PROBABILITY, AND STATISTICS	41
PATTERNS AND RELATIONSHIPS	
MATHEMATICAL SYSTEMS AND NUMBER THEORY DISCRETE MATHEMATICS	
GRADES 3 – 4	
NUMBER SENSE	83
GEOMETRIC AND SPATIAL SENSE	108
DATA ANALYSIS, PROBABILITY, AND STATISTICS	136
PATTERNS AND RELATIONSHIPS	152
MATHEMATICAL SYSTEMS AND NUMBER THEORY	165
DISCRETE MATHEMATICS	177
<u>GRADES 5 – 8</u>	
NUMBER SENSE	187
GEOMETRIC AND SPATIAL SENSE	213
DATA ANALYSIS, PROBABILITY, AND STATISTICS	
PATTERNS AND RELATIONSHIPS	
MATHEMATICAL SYSTEMS AND NUMBER THEORY	
DISCRETE MATHEMATICS	301
<u>GRADES 9 – 12</u>	
NUMBER SENSE	317
GEOMETRIC AND SPATIAL SENSE	329
DATA ANALYSIS, PROBABILITY, AND STATISTICS	359
PATTERNS AND RELATIONSHIPS	
MATHEMATICAL SYSTEMS AND NUMBER THEORY	
DISCRETE MATHEMATICS	429
Appendix A-DISCRETE MATH GRADE LEVEL STRANDS	451
Appendix B-COMMON ERRORS AS REPORTED BY MAP SCORERS	
Frror Reporting Form	461

NUMBER SENSE

K-2

1. Counting and grouping strategies.

Clarifications:

All students should know how to:

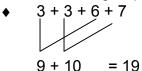
- Connect number words and numerals to quantities that they represent.
- Group and count objects by 2's, 3's, 5's, 10's etc.
- Choose appropriate grouping strategies to count larger groups of items.
- Count money i.e. 5's, 10's, 25's.

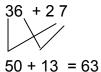
2. Mental computation and estimation strategies.

Clarifications:

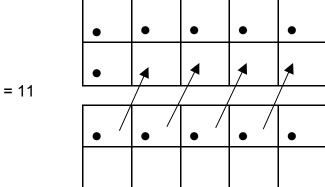
All students should know how to:

- Use mental computation strategies which could include:
 - ♦ Find compatible numbers (75 + 29 = 75 + 25 + 4) = 104
 - \bullet Adjust for nines (9 + 6 = 10 + 5); (29 + 45 = 30 + 45 1) = 74
 - Chunk or group known parts $52 + 48 \rightarrow 50 + 40 + (2 + 8) = 100$





• Visualize numbers using a ten-frame



6 + 5 = 11

- Determine a reasonable estimate to describe quantity (i.e. how many marbles in a jar).
- Determine a reasonable estimate to describe the length of an object (i.e. the length of a pencil.)

3. Place value.

Clarifications:

All students should know:

- How to read and write multi-digit numbers in standard form and words.
- How to represent numbers using models, and/or drawings.
- The magnitude (size) of numbers with emphasis on powers of 10 (hundreds, thousands).
- A numeral's value according to its place.
- How to recognize coins and bills.
- · How to identify the value of coins and bills.
- How to use appropriate money symbols.

4. Basic computation facts of addition, subtraction, multiplication, and division with whole numbers.

Clarifications:

All students should know:

- The meaning of addition/subtraction using real objects.
- How to represent quantities using written symbols for addition and subtraction.
- Multiple strategies for solving basic facts for addition (doubles, doubles + 1 or −1, counting on, adjust for nines).
- The zero property for addition and subtraction (6 + 0 = 6 and 6 0 = 6).
- The commutative property (2 + 3) = (3 + 2).
- The associative property (2 + 3) + 5 = 2 + (3 + 5).
- How to use inverse operations for addition and subtraction to solve problems (3 + 2) = 5 and (5 - 2) = 3.

5. Addition and subtraction of fractions with like denominators.

Clarifications:

All students should know:

- The meaning of fractions as a part of a whole (i.e. halves, fourths and thirds).
- The meaning of fractions as a part of a set.
- The meaning of mixed numbers.
- Fractional benchmarks of 0, 1/2, 1 (i.e. 1/3 is closer to 1/2 than 0).

6. U.S. customary and metric units of measure.

Clarifications:

All students should know:

- How to use a ruler, measuring cup and balance scale.
- How to estimate using an appropriate unit of measure.
- Which unit of measure is appropriate for the task:
- U. S. customary units of measure.
 - ♦ Distance: inch, foot, yard, mile.
 - Capacity: cup, pint, quart, gallon.
 - Mass: ounces, pounds.
- Metric units of measure.
 - Distance: centimeter, meter, kilometer.
 - ♦ Capacity: milliliter, liter, kiloliter.
 - Mass: grams, kilograms.

7. The appropriate use of calculators.

Clarifications:

All students should know how:

- To enter simple equations using the appropriate keys (numerals, operation signs, =).
- To determine whether a calculator generated answer is reasonable.
- To use the "constant" feature on the calculator for repeated addition, repeated subtraction, multiples, etc.
- To use a calculator to solve problems that require working with larger numbers than students' computational skills allow.
- To use a calculator to assist with problem solving.

Written Benchmark: A

Model, explore, develop, and explain number operations for whole numbers.

Problem 1:

Process Standards: 1.6 and 3.3

A train can be made with seven cubes using only two colors. How many different ways can you make two color trains using seven cubes? Record five or more combinations and write the number sentence that describes each train.

Solution Notes:

Students are given a quantity of cubes; enough to make at least five trains using the same two colors for all trains.

Example of Possible Answers:

6 + 1 =

4 + 3 =

2 + 5 = 7

2+1+2+1+1=7

3 + 4 =

Many other patterns are possible.

Prerequisites:

Students should:

- 1. Be able to count up to seven correctly (one to one correspondence).
- 2. Be able to write numbers through 7.
- 3. Be able to copy their train design.
- 4. Be able to write a number sentence.

Problem 2:

Process Standards: 1.6, 2.1, and 3.3

There are 12 eggs in a carton. I have to make one batch of pancakes and one batch of waffles. I need 4 eggs for the pancakes and 7 eggs for the waffles. How many cartons of eggs do I need? Use a picture and an equation to solve the problem. Write a sentence to explain your answer.

Solution Notes:



Pancakes

Waffles

4 eggs + 7 eggs = 11 eggs 12 eggs in a dozen – 11 eggs used = 1 egg left over.

Student explanations may say, "I need 1 carton because 4 + 7 = 11 and 1 dozen eggs are 12 eggs. I will have one egg left over."

Prerequisites:

Students should:

- 1. Know that a dozen equals 12 items.
- 2. Be able to demonstrate one to one correspondence.
- 3. Be able to write numbers through twelve.
- 4. Be able to make a pictorial representation of their solution.
- 5. Be able to express their thoughts in writing.
- 6. Be able to write an appropriate equation.

Written Benchmark: B

Use technology to explore numbers.

Problem 1:

Process Standards: 1.4, 1.6, and 2.7

It's time to practice counting backwards. Take your calculator and let's do a pattern. Enter the number 5. Press the (-) sign. Enter the number 1. Press the (=) sign. What's your number? Continue to press (=). Discuss the pattern. Then start at 10 independently and enter (-) and 1. Record your number each time as you enter (=). Write a sentence telling about your pattern.

Solution Notes:

9 8 7 6 5 4 3 2 1

I counted backwards when I subtracted 1.

Procedures may vary depending on calculator type.

Problem 2:

Process Standards: 1.4, 1.6, and 2.7

Read *Two of Everything* by Lily Toy Hong and discuss the patterns in the story. Each student needs a calculator and a hundreds chart. Have the students skip count by entering (+ 2) on the calculator. Color in the number (2) on the hundreds chart. Have students continue entering (=) and coloring in the numbers displayed. Discuss the visual and number patterns that appear on the hundreds chart.

Solution Notes:

Hundreds Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Possible solutions:

- 1. Every other number is colored.
- 2. All the colored numbers end in 2, 4, 6, 8, or 0. Note: Procedures may vary depending on calculator type.

Prerequisites:

Students should:

- 1. Recognize numbers to 100.
- 2. Know how to use a calculator for subtraction and skip counting.

Written Benchmark: C

Use physical models and real world experiences to construct number meanings.

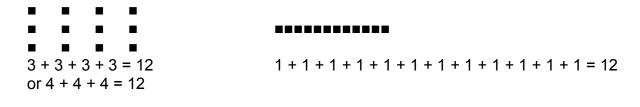
Problem 1:

Process Standards: 1.1, 1.8, and 1.10

Have students model how to arrange 6 cages in a pet store by having the students pretend to be animals and arranging themselves in front of the class. Students write down the number sentence describing the arrangement. The teacher explains to the children that they are the owners of a pet shop. There are 12 cages in the pet shop. Using cubes, have the student's model different ways to arrange the cages. Have them draw pictures and write number sentences to show some of the different arrangements possible. Have each child choose their favorite arrangement for the best display and tell why they chose that arrangement.

Solution Notes:

Examples of possible arrangements:



12 + 0 = 12

Possible Answer:

I like the circle arrangement because it is easier to walk around and view the animals. The animals can see each other.

Problem 2:

Process Standards: 1.10 and 3.3

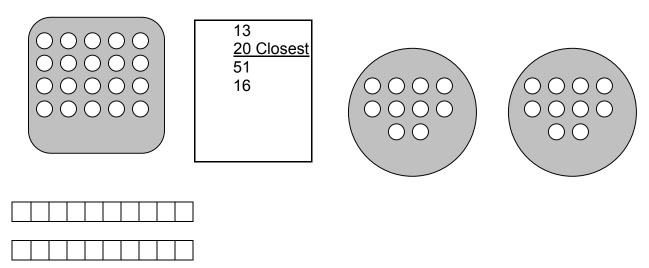
The teacher places a small quantity of candy or cereal (20 - 50 pieces) in a jar. Then the students estimate how many they think are in the jar. Record their answers on a large chart. The children and teacher then count the actual number in the jar and group the quantity into tens and ones. The children model the number with interlocking cubes into groups of tens and ones. They then review the chart of estimates determining which estimates were less, more, the same, or closest to the actual number of pieces of cereal or candy in the jar.

Solution Notes:

Answers will vary. Example based on 20 pieces.

What is your estimate?

Count into groups of 10



Written Benchmark: D

Demonstrate an understanding of our numeration system by relating counting, grouping and place value concepts.

Problem 1:

Process Standards: 3.6 and 4.1

The teacher will have the children remove their shoes and graph them on a large graph mat on the floor. Examples of kinds of shoes could include: tie shoes, slip-on shoes, velcro shoes. The teacher will discuss with the students the meaning of more, less and the same from the graph. Given a blank graph, the students will model the real graph using cubes and then they will color in the graph.

Solution Notes:

Answers will vary depending on data collected. Student comments would include: Most people have tie shoes. More people have velcro shoes than have slip-on shoes.

Graph with Cubes for Kinds of Shoes

 \mathcal{I} \mathbf{z} \sim \mathcal{I} \mathcal{I} 7 \mathcal{I} \sim \mathbf{z} \mathcal{I} \mathcal{I} \mathcal{I} \mathcal{I} \mathcal{I} SLIP-ON TIE VELCRO

Graph on Graph Paper

Х		
Х		
Х	Х	
Х	Х	
Х	Х	Х
Х	Х	Х
Х	Х	Х
TIE	VEL CPO	GI ID-UN

Problem 2:

Process Standards: 1.6, 1.7, and 3.3

I have a total of six base-ten blocks. Some of the blocks are tens and some are ones blocks. I want to represent a number that is less than sixty. What numbers can be represented using six blocks? Use your base ten blocks and draw pictures of each possible number. Write the number by each model. What are the largest possible number and the smallest possible number that can be made using six total blocks? How did you figure out the largest number and the smallest number?

Solution Notes:



Encourage children to find as many possible solutions as time permits. The largest number is 51 and the smallest number is 15. Other possible numbers are 42 and 24.

Possible Student Answer: I used as many tens as I could to get 51 and I used as many ones as I could for the smallest number.

Written Benchmark: E

Utilize number sense to develop number meanings and explore number relationships.

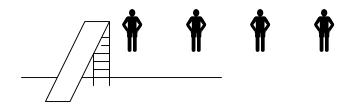
Problem 1:

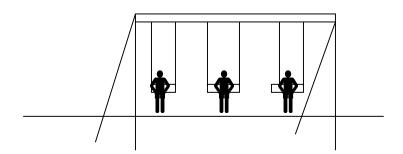
Process Standards: 1.8 and 3.3

Seven children are playing on the school playground. Four children are taking turns using the slide. How many children are not playing on the slide? Draw a picture and write a number story to show your solution.

Solution Notes:

4 + 3 = 7 There are three kids on the swings and four kids at the slide.





Proble i	m 2:
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Process Standard: 1.5

Put cards numbered 0 - 10 into a hat or box. Draw out two cards to show the children. Have the children construct a model of each number with interlocking cubes and compare the two models using words such as larger, smaller, greater than, and less than.

Solution Notes: I got 6 and 3.
6 is larger than 3.

Written Benchmark: F

Use a variety of mental computation and estimation strategies to solve specific problems.

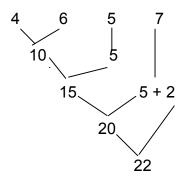
Problem 1:

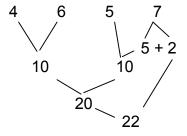
Process Standards: 1.6 and 3.3

Give pairs of student's four number generator cubes (dice). Ask them to roll the dice and add the numbers that appear on the topside of each of the four dice mentally. Have students take turns explaining to their partner how they were able to solve the problems mentally.

Solution Notes:

I rolled 4, 6, 5, and 7.





Problem 2:

Process Standards: 1.7, 1.8, and 3.3

Children take a handful of lima beans; estimate how many they have, and record there estimate. Then they count the number of lima beans they grabbed and record the actual number. Discuss with a partner how the estimate and the actual number of beans varied. Repeat this activity using cubes. Children will explain to a partner how they made their estimate the second time.

Solution Notes:

Students might say, "I can hold more lima beans because they are smaller, or my hand can't hold as many cubes because they are bigger."

Written Benchmark: G

Demonstrate an understanding of the attributes of length, capacity, weight, area, volume, time, temperature, and angle.

Problem 1:

Process Standards: 1.1 and 3.3

Brainstorm a variety of activities that the students think they can complete in one minute. Each group records three or four ideas from the list on individual cards. Have the group predict the sequence of the items according to which activity will take the least amount of time to the most amount of time. Do the activities and record the actual amount of time it took to complete each activity. Have the students compare their estimates with the results from doing the activities, and write sentences to tell how close or far apart the estimates and the actual time are to each other.

Solution Notes:

Possible Brainstorm Ideas:

Take a book to the library, Count to 100, Walk to the door, Jump 40 times, Write numbers to 50. Draw 30 stars.

Example of Possible Student Answer:

We chose that we could walk to the door in the least amount of time, count to 100 in the next least amount of time. Taking a book to the library would require the most time. When we timed the activities, we found that...

Problem 2:

Process Standards: 1.3 and 3.3

Give each group four small milk cartons and a scoop (laundry scoops work well). Have available a variety of objects to use to fill the milk cartons (beans, cubes, buttons, etc.). Ask the children to estimate how many scoops of each object it will take to fill their milk carton. Record the estimate on a record sheet. Have the students fill their milk cartons and record the actual number of scoops of each item it took to fill the milk carton. Have the students write a sentence describing how their estimate compares with the actual number of scoops. Students can estimate the milk cartons' weight and place them in order from heaviest to lightest. The students will use a balance scale to determine the heaviest to lightest.

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20	ILITIC	วท ท	iote	S

Possible Student Responses: Estimate of number of scoops: 3 Actual number of scoops: 5	BEARS
Estimate of number of scoops: 4 Actual number of scoops: 6	BEANS

I think the beans are lightest, then the cubes, then the buttons, and the bears will be the heaviest. When I weighed the containers I found out that \dots

Written Benchmark: H

Make and use standard and non-standard measurements in problems and everyday situations.

Problem 1:

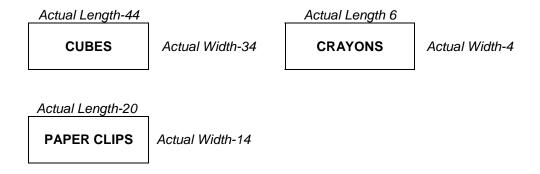
Process Standards: 1.2 and 1.7

Ask children to estimate the number of interlocking cubes it will take to find the length of their desk and the width of their desk. Each student will measure and record the length and width of their desks with the cubes. Then measure the length and width of their desks with other objects (crayons, pencils, paper clips, etc.) as non-standard units of measurement. With these objects the students should record both their estimates and the actual measurement. Have the students write sentences or draw pictures to explain why they might not have the same results with different objects.

Solution Notes:

Possible Student Responses:

I estimate that my desk is 42 cubes long and 38 cubes wide. I measured my desk and it is 44 cubes long and 34 cubes wide.



My answer for paper clips is bigger because paper clips are smaller than crayons.

Problem 2:

Process Standards: 1.2 and 1.7

Read *Jim and the Beanstalk* by Raymond Briggs. Discuss the giant's measurements and how they were found. Then lay out some pre-cut giant footprints. Have the students estimate how many cubes it will take to fill the giant's footprint. Work in groups to solve the problem. Have the students trace their foot on construction paper and cut it out. Have students estimate how many cubes it will take to fill the footprint and then fill the footprint with cubes. Write and/or draw pictures to show a comparison of the estimate to the actual count.

Solution Notes:

This activity shows non-standard units of measurement used for measuring area. Solutions will vary depending on the size of the cube used for measuring.

Possible Student Responses:

My estimate was 25 cubes and my count was 16 cubes. My estimate was too high.

Written Benchmark: I

Explore the concepts of fractions, mixed numbers, and decimals and be able to apply them to problem situations.

Problem 1:

Process Standards: 1.7 and 3.3

Read *Gator Pie* by Louise Mathews and discuss the story. Ask the students questions such as: Would you rather eat ½ or ¼ of the pie? Why would you rather eat one size rather than the other? If you are given a choice of ½ a pie or a whole pie, which one would you rather eat and why?

Solution Notes:

As students give answers flip through the book and display fraction pictures of $\frac{1}{2}$, $\frac{1}{4}$ and 1 or have fraction circles to display $\frac{1}{2}$, $\frac{1}{4}$ and 1 as students give answers. These visual models will help students understand the size of fractions.

Problem 2:

Process Standards: 2.1 and 3.3

Provide six snack foods such as apples or graham crackers to a group of four. Ask the group to share the snack evenly. Draw a picture of how your group shared the snacks so everyone's share was equal.

Solution Notes:

Each group member should take one whole and then break the remaining two graham crackers in half. The pictures should represent one whole and one half.

GEOMETRIC AND SPATIAL SENSE

K-2

1. Standard and non-standard units of measure.

Clarifications:

All students should know:

- Examples of nonstandard units of measurement.
 - Cubit (length from elbow to end of longest finger)
 - Pace (walking step) and foot to foot
 - Handspan, finger and knuckle to finger tip
 - Use a pencil, paper clips or links to measure a distance
 - Use water glasses to measure capacity
 - Use paper clips to measure weight
 - Use beans for length, area, capacity
- Examples of standard units of measurement.
 - English/U.S. customary system
 - -- Distance inch, foot, yard, mile
 - -- Capacity cup, pint, quart, gallon
 - -- Mass ounce, pound
 - Metric system
 - -- Distance centimeter, decimeter, meter, kilometer
 - -- Capacity liter, kiloliter
 - -- Mass gram, kilogram

2. Descriptions of two- and three-dimensional figures.

Clarifications:

All students should recognize:

- Two-dimensional figures (shapes) as having length and width with no depth. Examples:

 - **Triangles**
 - Circles
 - Rectangles including squares
- Three-dimensional figures (shapes) as having length, width, and height.

Examples:

- Cubes
- **Spheres**
- Cones
- Cylinders

- All students should know the meaning of:
 - ♦ Side
 - ♦ Vertices (points)
 - ♦ Symmetry
 - ♦ Congruency

3. Geometric shapes are found in the real world.

Clarifications:

All students should identify examples of geometric shapes in the real world.

- Two-dimensional figures:
 - Square hopscotch squares
 - ♦ Rectangle pieces of paper
 - ♦ Circle paper plate, clock face
- Three-dimensional figures:
 - ♦ Cube dice
 - ♦ Cylinder soda can
 - ♦ Sphere ball

4. Objects can be located by relative position.

Clarifications:

All students should know relative positions might be:

- Points on a graph
- Numbers on a line
- Ordinal numbers
- Position words such as above, below, etc.

5. The process of measurement.

Clarifications:

All students should know how to:

- Use consistency when working with standard or nonstandard units of measurement.
- Repeat/count units of measurement.
- Measure in different forms of measurement including:
 - ♦ Length
 - ◆ Time
 - ♦ Capacity
 - ♦ Mass

- ♦ Area
- ♦ Volume
- ♦ Temperature
- Use time concepts including:
 - Calendars
 - ♦ Hours
 - One-half hours
 - Fifteen minute intervals
 - Elapsed time
 - Seasons

Written Benchmark: A

Describe, model, draw, and classify shapes.

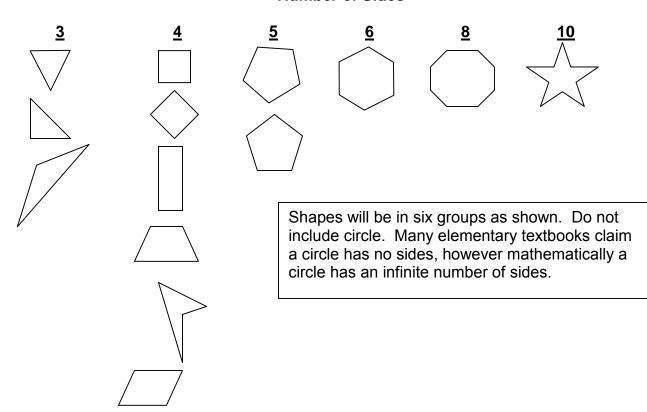
Problem 1:

Process Standard: 1.6

Given a set of 12 or more shapes, group them according to the number of sides.

Solution Notes:

Number of Sides



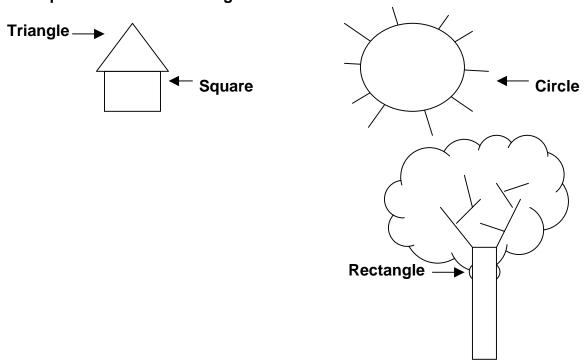
Problem 2:

Process Standards: 1.8 and 2.1

Draw or construct a picture using the basic shapes of circle, rectangle, square, and triangle. Identify and label at least one example of each shape.

Solution Notes:

Examples of Student Drawings:



The basic shapes will be labeled accurately to show identification of each shape used in the completed picture.

Written Benchmark: B

Investigate and predict the results of combining, subdividing and changing shapes.

Problem 1:

Process Standards: 1.6 and 2.1

Ask the children to make a triangle on their geoboard. Have them investigate changing the triangle to a rectangle, square, or other geometric shape. Explain how their shape changed and shows the attributes of a square, etc.

Solution Notes:

Children should correctly show a triangle on their geoboard. They should move the geoband to show at least two other shapes. The explanation should demonstrate knowledge of the attributes of each shape shown.

Written Benchmark: C

Visualize, draw and compare shapes.

Problem 1:

Process Standard: 2.1

Use modeling clay to construct a cube, a sphere, a cone, and a cylinder. Identify each solid (3-dimensional) shape. Then use the shapes to construct an object such as making the cone into a tree, the cylinder into an animal, etc. Compare the solid (3 - dimensional) shape to the object.

Solution Notes:

The constructed results will be a cube, a sphere, a cone, and a cylinder, which are changed into a tree, an animal, a present, etc.





Cylinder

Prerequisites:

Students should:

1. Be able to accurately identity three-dimensional shapes.

Written Benchmark: D

Connect geometric ideas to number and measurement ideas.

Problem 1:

Process Standard: 4.1

Given 4 to 5 cylindrical items, students will identify which holds less than a liter and which holds more than a liter.

Possible cylindrical objects:

- A soda can
- A bucket (with straight sides)
- A glass gallon jug
- A coffee can

Solution Notes:

The answers will vary according to the items selected.

Prerequisites:

Students should have:

1. Prior knowledge of the size of a liter.

Written Benchmark: E

Explore geometry in their world.

Problem 1:

Process Standards: 1.8, 1.10, and 2.1

Students will work in pairs. The class will take a math walk through the school and outside of the building. Students will look for objects that have geometric shapes (circle, square, rectangle, and triangle) in them or shapes that are made up of geometric figures. Students will record what they find by describing the object or drawing and labeling pictures. After the class has returned to the classroom, students will share their findings, including the attributes of each shape. A class table or chart could be constructed showing at least one contribution from each pair of students.

Solution Notes:

Answers will vary. Responses should include at least two attributes describing each shape.

Prerequisites:

Students should:

1. Be able to identify a circle, square, rectangle, and triangle.

Written Benchmark: F

Investigate concepts of lines, angles, similarity, congruence, and symmetry.

Problem 1:

Process Standard: 2.1

Using pattern blocks, students work in pairs. One student builds a design (on half of a two-part mat) and the other student builds an identical design (congruent) or the mirror of the design creating a line of symmetry.

Solution Notes:

Teacher will observe the designs built.

Geometric and Spatial Sense What All Students "Should Do" K – 2

Written Benchmark: G

Investigate length, capacity, weight, mass, area, volume, time, and temperature.

Problem 1:

Process Standards: 1.8 and 2.1

Read *The Grouchy Ladybug* by Eric Carle to the class. Have the students draw pictures of their daily activities. Choose four times of the day to have the students illustrate, i.e. 7:30 a.m., 9:00 a.m., 12:15 p.m. and 4:00 p.m. Students will write a sentence to go with each picture explaining what they are doing at that time of day.

Solution Notes:

Answers will vary.

Problem 2:

Process Standards: 1.8 and 2.1

Read *How Big Is a Foot*? by Rolf Myller to the class. Have the student's pair up and trace around each other's foot. Each student will cut out his or her paper foot. Using the paper foot, each student will measure designated items in the classroom and record their findings.

Solution Notes:

Students will share their results in class discussion.

Students may report: "The top of my desk is 3 of my feet."

Geometric and Spatial Sense What All Students "Should Do" K – 2

Written Benchmark: H

Use standard and non-standard units of measure.

Problem 1:

Process Standard: 1.2

Students will measure the length of their desktop with an unsharpened pencil and record their finding. Then each student will predict how many inches in length his or her desktop measures. Students will measure the desktop with a ruler and will compare the actual measurement to their prediction by writing a sentence. Other items that could be used are large paper clips, standard size erasers, and capped markers.

Solution Notes:

Student sentences should state the predicted measurement, the actual measurement, and how they compare.

Geometric and Spatial Sense What All Students "Should Do" K – 2

Written Benchmark: I

Locate objects by relative position including top, bottom, left, right, over, and under.

Problem 1:

Process Standard: 2.1

Given the seven-tangram shapes, each student will create a design by tracing around each shape. Students will pair up. Hiding their tangram design, one person will give oral directions to their partner describing how the tangram pieces should be placed on the paper to recreate the hidden tangram design.

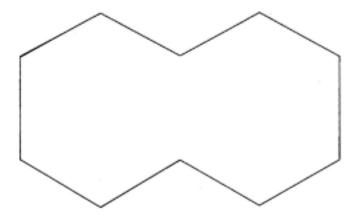
Solution Notes:

Students should correctly use the terms: top, bottom, left, right, over and under as they describe their design to another child.

Problem 2:

Process Standards: 1.6 and 2.1

Given pattern blocks, ask students to cover the shape below using two blocks, four blocks, and six blocks. Ask students to draw and describe their solutions.



Solution Notes:

Answers may vary. Possible combinations are:

- 2 Blocks 2 yellow hexagons,
- 4 Blocks 1 yellow hexagon, 1 red trapezoid, 1blue rhombus, 1 green triangle,
 - 4 red trapezoids
- 6 Blocks 3 red trapezoids, 3 green triangles,
 - 1 yellow hexagon, 1blue rhombus, 4 green triangles

Students should describe their solutions correctly using the terms: top, bottom, left, right, over and under.

DATA ANALYSIS, PROBABILITY, AND STATISTICS

K-2

Data Analysis, Probability, and Statistics What All Students "Should Know" K – 2

1. Strategies to collect data.

Clarifications:

All students should know strategies to collect data.

Examples:

- Participate in surveys
- Select data from teacher provided information and objects
- Gather objects (leaves, stuffed animals, buttons...) and information from their environment
- Gather information from graphs, charts, and tables

2. Strategies to organize data.

Clarifications:

All students should know strategies to organize data.

Examples:

- Sort, categorize, group
- Count
- Tally
- Use charts, tables, lists
- Use grids
- Use Venn diagrams

3. Different ways of displaying data.

Clarifications:

All students should know different ways to display data.

Examples:

- Complete—
 - ♦ Tables and charts
 - Pictographs
 - Bar graphs
- Use manipulatives to create graphs

4. The appropriate display of data.

Clarifications:

All students should know:

- Different kinds of graphs are used to display a variety of data
- One type of graph may do a better job of conveying information than another type of graph for a particular set of data

5. The appropriate use of technology.

Clarifications:

The students should know appropriate use of available technology to collect, organize, and display data:

- Computer software
- Internet
- Electronic Media
- Calculator
- Overhead projector

Data Analysis, Probability, and Statistics What All Students "Should Do" K – 2

Written Benchmark: A

Collect, organize, and describe data through the use of technologies and other resources.

Problem 1:

Process Standards: 1.1, 1.3, and 1.8

After taking a nature walk where each student collected two to three leaves, have the students generate ways to sort the leaves such as shape, size, or color. Vote to determine which method to use for the class sorting process. Create a chart to record the data. Have students place their leaves in the correct group, and place a tally mark on the chart in the appropriate category.

Have students describe how the data was categorized. Find which category has the greatest number of leaves, the least number of leaves, and the total number of leaves. Ask students what other items could be found in nature that could be sorted and charted in a similar manner.

Solution Notes:

Teacher observation of the way students categorizes the leaves and charts the results.

Data Analysis, Probability, and Statistics What All Students "Should Do" K – 2

Written Benchmark: B

Construct, read, and interpret displays of data through verbal, non-verbal, symbolic, and graphic forms.

Problem 1:

Process Standard: 1.8

Have each student bring one piece of fruit. On a large sheet of paper, make a blank graph and label it with each type of fruit that students brought to school. Have the students place their fruit in the appropriate place on the graph. Discuss which kind of fruit most students brought to school and the least kind of fruit brought to school by students, etc.

Solution Notes:

The teacher will observe the way the students sort fruit and graph the results.

Problem 2:

Process Standard: 1.8

Given $\frac{1}{4}$ cup of cereal or candy containing different colors, students will sort and graph by colors. Make a picture graph of their results.

Solution Notes:

The student answers will vary based on their sample of cereal or candy.

			CERE	EAL	OR CAND	Y GRAPH			
N		6							
U M		5							
B E	С	4							
E R	O	3							
0	L O	2							
F	R	1							
E A C H			Re		Yellow quals one	Orange	Green ereal or Ca	Blue andy)	Purple

What color has the most? What color has the least?

Eat three of your pieces of cereal or candy. Explain to a partner how this would change your graph.

Prerequisites:

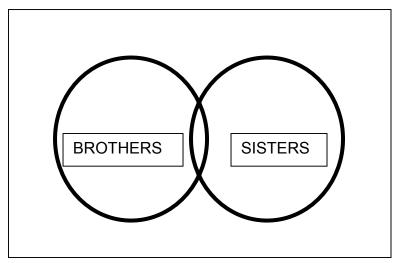
Students should be able to:

- 1. Identify the different colors.
- 2. Know how to count.
- 3. Have knowledge of picture graphs.

Problem 3:

Process Standards: 1.8 and 4.1





Have children in the class write their names in the appropriate place on the Venn diagram answering the question. Use inch graph paper to have students create, label, and title a bar graph to display data from the Venn diagram. Students should answer the following questions: What category has the most students? Do any of the categories have an equal number? Are any of the categories empty? Explain your graph to a friend. Be sure to make comparisons between the groups.

Solution Notes:

The teacher will observe as the students complete the Venn diagram, analyze the data, make the bar graph and answer the questions. The answers will vary determined by the make-up of the class.

Data Analysis, Probability, and Statistics What All Students "Should Do" K – 2

Written Benchmark: C

Solve problems that require collecting and analyzing data.

Problem 1:

Process Standards: 4.1 and 3.4

Have the students survey their family members to discover the favorite candy bar of each person. Construct a table to display the class data. As a class, discuss which kind of candy bar is liked the most, least, etc. Have students imagine they are responsible for stocking a concession stand. As a class or individually, have the students discuss or write which three to five kinds of candy bars they would choose and tell their partner why they made those selections.

Solution Notes:

The students' answers will vary determined by the data collected.

Problem 2:

Process Standards: 3.4 and 4.1

Look at the graph to answer the following questions.

- 1. How many more visitors were at the zoo on Friday than on Wednesday?
- 2. How many visitors were at the zoo this weekend?
- 3. Write in your journal why there were more visitors on Sunday and Saturday.
- 4. How would this data help the zoo managers make decisions?

Solution Notes:

- 1. 45 visitors
- 2. 155 visitors
- 3. Answers will vary.
- 4. Answers will vary.

Example: More employees will be required on Saturday; less concession foods will be needed on Wednesday.

Prerequisites:

Students should know:

- 1. Skip counting by five by ten.
- 2. How to analyze a pictograph.

Data Analysis, Probability, and Statistics What All Students "Should Do" K – 2

Written Benchmark: D

Explore concepts of chance.

Pro	b	lem	1	:

Process Standard: 1.6

Use the following list of activities and games relating them back to chance. What is the probability of ?

- Place three green counters and one yellow counter in a bag. Draw out one object counter, record the color on a tally chart, and place the counter back in the bag. After ten samplings, predict the contents of the bag. Continue with ten more samples. Discuss the prediction and make changes if desired. Check the contents of the bag to verify the results.
- 2. Colored spinners. Make spinners to increase the chance of getting one color over the other colors (i.e. ¼ blue, and ¾ red).
- 3. Number cubes:
 - Toss a cube and record the number showing on top of the cube. Repeat this
 procedure several times and compare the data.
 - Toss two cubes and record the sum of the numbers on the two cubes.
 Record the data. Repeat the procedure several times and analyze the data by comparing the number of even sums to the number of odd sums.
- 4. Games: Play games like Old Maid or Go Fish.
- 5. Penny Toss: Toss a penny, record the number of times heads or tails comes up.

Solution Notes:

1. Green	Yellow		2. Blue	Red	
JJH I	IIII	10 draws	III	INI II	10 draws
4M III	II	10 draws	IIII	KĮI I	10 draws
14	6	20 draws	7	13	20 draws

Prerequisites:

Students should:

- 1. Know how to keep track of data using tally marks.
- Have a basic knowledge of probability and counting.

PATTERNS AND RELATIONSHIPS

K-2

Patterns and Relationships What All Students "Should Know" K – 2

1. Mathematical ideas may be represented with visual models.

Clarifications:

All students should know:

- Left to right progression.
- Shape recognition.
- Sorting and classifying by attribute techniques, i.e. color, texture, sound, shape, number, and size.
- How to use manipulatives in creating and extending patterns.

2. Mathematical symbols can be used to represent real-world situations.

Clarifications:

All students should recognize:

- Geometric shapes are found in patterns in cloth, quilts, buildings, etc.
- Pictures and numbers may represent how many of real life item are in the set.
- Symbols such as +, -, =, <, and > can be used with numbers to represent a comparison or an event.

3. Patterns and relationships can be represented in a variety of ways.

Clarifications:

All students should know patterns and relationships found in:

- Sound/rhythm, i.e. clap-stomp, musical notes, etc.
- Physical objects, i.e. desk chair desk chair
- Pictorial representations, i.e. cat dog cat dog (in pictures)
- Symbols $\leftarrow \uparrow \rightarrow \downarrow$ $\nabla \Box O \nabla \Box O$
- Dog cat mouse dog cat mouse
- Quantity, i.e. 2, 4, 8, 16
- ABBABB

4. Information can be organized to look for a pattern or relationship.

Clarifications:

All students should know how to:

- Represent a pattern.
- Use a chart or graph.

5. Patterns can be geometric and/or numeric.

Clarifications:

All students should recognize patterns with:

- Geometric figures (shapes), i.e. triangle, square, triangle, square...
- Rotating objects or symbols, i.e. \leftarrow , \uparrow , \rightarrow , \downarrow
- Numerical patterns, i.e. counting by two's even numbers, counting by five's, counting by 10's, counting by 100's.

Patterns and Relationships What All Students "Should Do" K – 2

Written Benchmark: A

Create, recognize, describe and extend a wide variety of patterns.

Problem 1:

Process Standards: 1.6, 2.1, and 3.3

The teacher will arrange students in a boy-girl-boy-girl pattern. Then discuss with the students identifying attributes used in the pattern. Have students suggest and model ways of varying the pattern. The teacher will then arrange a model sound pattern, i.e. clap-stamp-clap-stamp. Students repeat the pattern. Have students demonstrate and model a variation of the pattern. Finally the teacher will use blocks, cubes, tiles, or beads, to create a repeating pattern. Students will extend and explain the pattern orally. Students can extend patterns begun by another student.

Solution Notes:

The teacher should:

1. Observe the completed and created patterns.

Patterns and Relationships What All Students "Should Do" K – 2

Written Benchmark: B

Represent and describe mathematical relationships.

Problem 1:

Process Standard: 1.6

- The teacher will use a hundreds chart to have the class find mathematical patterns. The class will discuss the reasoning for the patterns found.
- The students will find and identify different patterns using individual hundreds charts. The patterns can be identified by using different colors to illustrate the pattern.
- The students will use a hundreds chart to color in the 2's, 5's, and 10's using a different color for each pattern.
- The students will draw a square around common multiples i.e. 2, 4, 6, 8,

5, 10

10

• The student will use a hundreds chart to color in the numbers needed to find the total number of wheels on ten motorcycles. The student will explain the pattern.

Solution Notes:

Students will find that there are 20 wheels on the 10 motorcycles possibly by coloring in every other number on a hundreds chart. Student explanations may include statements such as: I counted by even numbers; I counted by 2's, or other valid explanations.

Patterns and Relationships What All Students "Should Do" K – 2

Written Benchmark: C

Investigate the use of variables and open sentences in expressing relationships.

Problem 1:

Process Standards: 1.6, 1.8, and 3.3

Student will use the constant function on a calculator and complete the table to find out how many total legs for 5 dogs.

DOGS	1	2	3	4	5
LEGS	4				

How many total legs for 7 dogs? How many total legs for 10 dogs?

Make a chart from folded construction paper. Glue on pictures of cats: 1 in 1st box, 2 in 2nd box, etc. to 4. Each student will then complete the chart, filling in the correct number of eyes under each picture.

Solution Notes:

The teacher will observe the student participating in the open sentence activities. The teacher will evaluate the charts:

DOGS	1	2	3	4	5
LEGS	4	8	12	16	20

The cat pictures will be pasted on correctly and the eyes of the cat drawn in correctly.

CATS				
	• •			
EYES				
	2 eyes	4 eyes	6 eyes	8 eyes

Discuss how many eyes should be drawn under the picture of 2 cats. Students should complete the chart by drawing eyes under the picture of cats. How many eyes do 3 cats have? How many eyes do 4 cats have? Explain the counting pattern to a friend.

MATHEMATICAL SYSTEMS AND NUMBER THEORY

K-2

Mathematical Systems and Number Theory What All Students "Should Know"

K-2

1. Basic operations of addition, subtraction, multiplication and division are related to each other.

Clarifications:

All students should know:

- Addition/subtraction vocabulary.
- Manipulatives may be used to develop concepts of basic operations.
- Addition and subtraction are inverse operations.
- Commutative property of addition (sum is not changed by the order of the addends).
- Fact families.
- Addition may be used to check subtraction.
- Multiplication can be solved or modeled using repeated addition or arrays.
- Equal groups explain multiplication.
- Equal sharing explains division.

2. The concepts of factors and multiples in relation to multiplication and division.

Clarifications:

All students should know:

- Skip counting is a preparation of multiplication.
- How to use money for skip counting.
- Tally counting techniques.
- Double patterns i.e. 2 + 2, 3 + 3.
- Fractional parts of a set, i.e. half of four objects is two.

Mathematical Systems and Number Theory What All Students "Should Do" K – 2

11 2

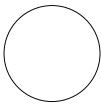
Written Benchmark: A

Develop the need for whole numbers, integers and rational numbers, including fractions and decimals by looking for patterns and relationships to solve problems.

Problem 1:

Process Standards: 1.6, 3.2, and 3.3

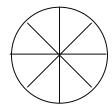
Sue had three friends over for dinner. They shared one pizza. The pizza was cut into 8 equal pieces. In the circle below show how the pizza was divided.



How many pieces of pizza will each person get to eat?

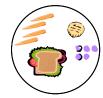
Jane and Sherry are going on a picnic. In their basket they have two sandwiches, 8 carrot sticks, 10 grapes, and 2 cookies. On the plates below show how the food can be divided equally.





Two pieces for each person.





One sandwich, 4 carrot sticks, 5 grapes, and 1 cookie.

Mathematical Systems and Number Theory What All Students "Should Do" K – 2

Written Benchmark: B

Develop and use number operations and order relations for decimals.

Problem 1:

Process Standards: 1.6, 3.2, and 3.3



Think of different ways to spend 25¢. Give at least 4 possible answers.

Playing in games of 2 or 4 players. Using a spinner or number cube, students will take the number of pennies equal to the number shown on the spinner or number cube: 5 pennies can be exchanged for one nickel. 2 nickels can be exchanged for one dime. Two dimes and one nickel can be exchanged for one quarter.

Solution Notes:

Possible answers-

1 book, 3 pencils Game: The first player to get 4 quarters (\$1.00) wins.

2 balls, 1 pencil

1 ball, 1 book, 1 pencil

1 kite

1 ball, 3 pencils

5 pencils

2 books, 1 pencil

Prerequisites:

Students should know:

- 1. Money values for penny, nickel, dime, and quarter.
- 2. How to skip count.
- 3. How to add.
- 4. How to count money.

Mathematical Systems and Number Theory What All Students "Should Do" K – 2

Written Benchmark: C

Develop an understanding of how basic arithmetic operations are related to one another.

Problem1:

Process Standard: 1.6

Use manipulatives and numbers to complete the chart:

Start with:	Add:	Total:	Subtract:	Number left:
2	8		8	
5	5		5	
4	6		6	
3	7		7	
1	9		9	
'	9		9	

Explain to a friend the pattern that you see on the chart.

Then give each student a bag with 8 counters. They reach into the bag and take out some counters. On paper they record the number sentence that this action represents, i.e. 8 - 3 = 5. When the counters are put back in the bag, what number sentence is represented, i.e. 3 + 5 = 8.

Solution Notes:

The pattern in the chart represents the "10" fact families. The teacher will observe the students as they complete the chart.

Total	
10	
10	
10	
10	
10	

Number left:
2
5
4
3
1

Prerequisites:Students should know:

1. How to add/subtract.

Mathematical Systems and Number Theory What All Students "Should Do"

K-2

Written Benchmark: D

Develop and use number theory concepts, including factors and multiples in problem solving.

Problem 1:

Process of Standards: 1.6 and 3.5

There are 14 feet in Mike's house. Mike has a pet cat. How many people are in Mike's family?

Carol has 3 horses. Each horse needs 2 scoops of grain each day. How many scoops of grain does Carol need each day?

Solution Notes:

Five (5) or an illustration explaining the answer. Six (6) or an illustration explaining the answer.

Prerequisites:

Students should know:

- 1. Rational thinking strategies such as, 2 feet for each person, 4 feet for each cat.
- 2. Problem-solving strategies.
- 3. How to skip count.
- 4. The addition and subtraction facts.

Mathematical Systems and Number Theory What All Students "Should Do"

K-2

Written Benchmark: E

Model, develop, and explain basic facts and algorithms with reasonable proficiency.

Problem 1:

Process Standards: 3.4 and 3.6

Alex had twelve gumballs. He wanted to share one-half of them with his sister. He gave her four gumballs. Did he give her the right amount? Explain why the answer is correct or incorrect.

Solution Notes:

No, because 4 is not one-half of 12; illustration explaining the answer.

Problem 2:

Process Standards: 3.4 and 3.6

25¢







Sam and Jim went to the school's store. Sam needed to buy school supplies. Sam has \$1.00 to spend. He wants to buy 1 box of crayons, 2 notebooks, 2 pencils, and 1 eraser. Jim doesn't think that Sam has enough money to buy these items. Sam thinks he does have enough money. Who is correct, Sam or Jim? Explain your answer.

Solution Notes:

Sam is correct.	1 box of crayons	25¢
	2 pencils	20¢
	2 notebooks	50¢
	1 eraser	5¢
	\$1.00	or 100¢

Prerequisites:

Students should know:

- 1. Problem-solving strategies.
- 2. How to add money.
- 3. If an answer is reasonable.

DISCRETE MATHEMATICS K-2

Discrete Mathematics

Discrete Mathematics can be more easily interpreted by thinking in five categories.

- Networks and Pathways
- Counting combinations/permutations
- Fair Division
- Logic and Reasoning
- Matrices

The clarifications for the knowledge statements are related to these five categories.

K-2

1. Numbers in sequence to count objects.

Clarifications:

All students should recognize:

- 1 to 1 correspondence
 - Sorting by attributes: color, size, shape
 - Equal groups
 - Venn diagrams
- Ordinal numbers: 1st, 2nd, and 3rd...
- Skip counting
- Number arrangements to equal the sum of parts
 - Combinations of number to create algorithms
 - Venn diagrams

2. Definition of "more" and "fewer".

Clarification:

All students should:

 Recognize which quantity or set has more or fewer when comparing two quantities or sets.

3. Definition of "same" and "different".

Clarifications:

All students should know how to:

- Categorize and sort objects by 1-2 attributes.
- Recognize when a quantity or set is the same or different when comparing two quantities or sets.

4. Definition of "shortest" and "longest".

Clarifications:

All students should know how to:

- Compare and sort objects by length.
- Order objects by length.

Written Benchmark: A

Determine what should be counted in a set of objects and actually count the objects.

Problem 1:

Process Standards: 1.8 and 3.5

There are 3 boys and 3 girls at a birthday party. If they line up boy, girl, with the birthday boy being first in line, would the 4th child be a boy or a girl? Explain your answer using pictures, numbers, and a sentence.

Solution Notes:

1	2	3	4	5	6				
В	G	В	G	В	G				
	The	The 4 th person is a girl.							
G	В	G	В	G	В				
6	5	4	3	2	1				
	The 4 th person is a girl								

Written Benchmark: B

Predict whether the set contains more or fewer of one subset than the other.

Problem 1:

Process Standards: 1.2 and 2.1

Give students two baggies of counting materials (i.e. beans, blocks, coins, etc.). Ask students to record a prediction for the number of items in each baggie. Then have students count the items in each baggie. Write a statement to compare their prediction and the actual count of each baggie. Write a second statement describing which baggie has more.

Solution Notes:

Statements should reflect a comparison between the prediction and actual count. Statements should use the words more or fewer to describe the quantities.

Written Benchmark: C

Illustrate or explain how the subsets of objects are the same or different.

Problem 1:

Process Standards: 1.6 and 4.1

Give the students a set of manipulatives (i.e. buttons or attribute blocks) representing several attributes. Have the student's sort and categorize the objects by 1 to 3 attributes. Draw a picture or write a sentence to explain the rules used to sort and categorize the objects.

Solution Notes:

Students should:

1. Have the objects sorted logically according to their rule, and clearly explain the attributes used in the rule.

Written Benchmark: D

Identify and discuss overlapping subsets of objects (Venn diagrams).

Problem 1:

Process Standards: 1.6 and 3.5

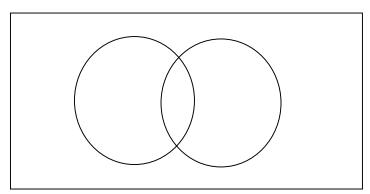
Whole Class Activity—Take a poll of your classmates. Then place your classmates' names in the Venn diagram. Be sure to label the diagram.

Class Poll: Who likes oranges?

Who likes apples?

Who likes oranges and apples?

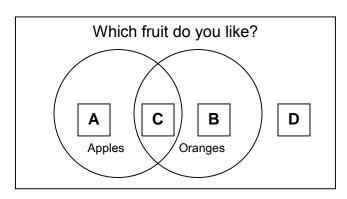
Who doesn't like either oranges or apples?



Solution notes:

Who likes oranges?------B
Who likes apples?------A
Who likes apples and oranges?------C
Who likes neither apples or oranges?----

Which fruit do you like?



Written Benchmark: E

Create algorithms based on constructing meaning from explorations.

Problem 1:

Process Standards: 1.5 and 3.3

Five classrooms won ice cream treats for good attendance. Each class had 19 students. Students will work with a partner to find how many ice cream treats are needed in all. Solutions could be recorded on chart paper. In a class discussion, partners should explain their solution.

Solution Notes:

Children may create many ways of exploring with the addition of five groups of 19. Observe how students take apart and recombine the number 19. Some may adjust one of the 19's to make the others 20 giving them the number sentence: 20 + 20 + 20 + 20 + 15 = 95. Others may say 19 is almost 20 and think 20 + 20 + 20 + 20 + 20 = 100 Then they could remove 1 for each 19 counting back to 95. Accept any explanation that can be justified.

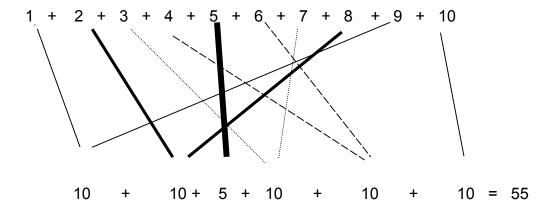
Problem 2:

Process Standards: 1.5 and 2.1

Read *Bat Jamboree* by Kathi Appelt. In this story, the children encounter bats having a jamboree. First one bat performs, then two bats, and so on until finally a group of 10 bats are enjoying the jamboree. Discuss how many total bats are in the story. Ask the students to devise a method for finding the total number in the story. *(One Hunter* by Pat Hutchins is a tale that follows a similar pattern.)

Solution Notes:

Some children may list the numbers as 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10. Some may notice the combinations of 10 that exist and add using a chunking method.



Written Benchmark: F

Determine a path through a maze, whether a street network could be traveled going over each street one time, and the shortest distance traveling on a network of roads or streets.

Problem 1:

Process Standards: 1.8 and 2.1

The class will identify different places in and out of the school building that they walk to each day (i.e. gym, lunchroom, playground). Working in pairs the students will answer What is the shortest path from our classroom to the lunchroom? these questions:

What is the longest path from our classroom to the playground?

What is the shortest path from our classroom to the gym?

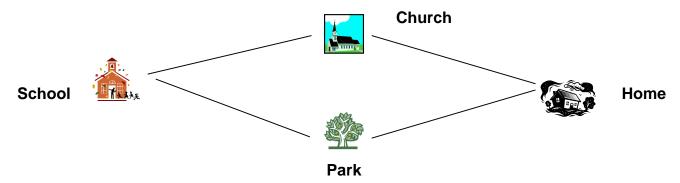
Each pair will record their solutions using drawings and/or words on chart paper. Solutions will be shared in a class discussion.

Solution Notes:

Accept any valid answer.

Problem 2:

Process Standards: 1.8 and 2.1



Looking at the drawing of the pathways above, how many different ways could a student walk home from school?

Solution Notes:

Students should identify two pathways for traveling between School and home:

- 1. School, past the Church, and then Home.
- 2. School, past the Park, and then Home.

Written Benchmark: G

Apply the concept of fair division to real-world situations.

Problem 1:

Process Standards: 2.1 and 3.3

A student brought a large bag of animal crackers to school for a class treat. "How will we know how much each person should receive?" the teacher asks.

Students could work with a partner to come up with a solution. Each pair must tell why their solution is fair.

Solution Notes:

Students must have a solution and a reasonable explanation for why their solution is fair.